# Dependency Trees, Permutations, and Quadratic Assignment Problem

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## Categories and Subject Descriptors

I.2.6 [**Artificial Intelligence**]: Learning; I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search; G.1.6 [**Numerical Analysis**]: Optimization

## General Terms

Algorithms

## Keywords

Estimation of distribution algorithm, dependency tree, permutation domain, quadratic assignment problem, deceptive ordering problem, linkage learning.

Since in many important real world problems—such as the quadratic assignment problem, scheduling, and vehicle routing—candidate solutions can be represented by permutations, designing efficient optimization techniques for solving problems defined over permutations is an important challenge. This paper describes and analyzes an estimation of distribution algorithm based on dependency tree models (dtEDA), which can explicitly encode probabilistic models for permutations. dtEDA is tested on deceptive ordering problems and a number of instances of the quadratic assignment problem (QAP). The performance of dtEDA is compared to that of the standard genetic algorithm with the partially matched crossover (PMX) and the linear order crossover (LOX). In QAP, the robust tabu search (RTS) is also included in the comparison.

Two approaches have been typically used for solving permutation problems with estimation of distribution algorithms (EDAs): (1) Real-valued EDAs with random keys and (2) EDAs with explicit permutation models. Here we consider the latter approach because we believe that the redundancy of random-key encoding has negative effects on efficiency of permutation-based EDAs and the focus should be put on designing probabilistic models that address specific types of regularities in typical permutation-based problems.

The probabilistic model used in dtEDA to model selected solutions and sample new solutions is a standard dependency tree (DT)—it is a directed acyclic graph where one node (the root) does not have any parents and all other nodes have exactly one parent. We chose DTs because of two main reasons: (1) DTs are capable of encoding dependencies between different permutation elements but (2) DTs are still simple enough for efficient learning with relatively small samples. The dependency tree is built from the population of selected permutations using the scoring metric based on mutual information between connected variables and any graph algorithm for finding the maximum spanning tree (e.g. Prim's algorithm). To generate only valid permutations (instead of arbitrary vectors), the sampling is modified so that when each variable is sampled, it is only allowed to obtain a value that has not yet been generated.

The results on the deceptive ordering problems indicate that dtEDA significantly outperforms GA with both PMX and LOX on the absolute ordering problem. On the other hand, on the relative ordering problem, dtEDA performs similarly to GA with PMX but both these methods are outperformed by GA with LOX.

On QAP instances from QAPLIB, most results indicate that RTS performs better than dtEDA, GA with PMX, and GA with LOX. These results are in agreement with the hypothesis that most publicly available random QAP instances do not contain much structure and represent a challenge for metaheuristics that exploit some form of the problem structure. Nonetheless, in the most difficult, structured QAP instances, dtEDA and GA with PMX perform best. In almost all cases, dtEDA outperforms all GA variants.

In addition to standard QAP instances from QAPLIB, we applied all compared methods to the QAP instances created by transforming the microarray placement problem to QAP. In this case, dtEDA obtained results that were better than any previously published results.

More details on this work can be found in Pelikan, M., Tsutsui, S., & Kalapala, R. (2007), *Dependency Trees, Permutations, and Quadratic Assignment Problem*, MEDAL Report No. 2007003, Missouri Estimation of Distribution Algorithms Laboratory, University of Missouri, St. Louis, MO. The report can be downloaded at
`http://medal.cs.umsl.edu/files/2007003.pdf`

## Acknowledgments

This project was sponsored by the National Science Foundation under CAREER grant ECS-0547013, by the Air Force Office of Scientific Research under grant FA9550-06-1-0096, by the *VolkswagenStiftung* (Germany) within the program "Nachwuchsgruppen an Universitäten", and by the University of Missouri in St. Louis through HPCC sponsored by Information Technology Services. The U.S. Government is authorized to reproduce and distribute reprints for government purposes notwithstanding any copyright notation thereon. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSF, AFOSR, or the U.S. Government.


*GECCO'07,* July 7–11, 2007, London, England, United Kingdom.
ACM 978-1-59593-697-4/07/0007.