Indirect Co-evolution for understanding Belief in an Incomplete Information Dynamic Game

Nanlin Jin
Department of Computer Science, University of Essex, Colchester, CO4 3SQ UK
njin@essex.ac.uk

ABSTRACT

This study aims to design a new co-evolution algorithm, *Mixture Co-evolution* which enables modeling of integration and composition of *direct co-evolution* and *indirect co-evolution*. This algorithm is applied to investigate properties of players' belief and of information incompleteness in a dynamic game.

Categories and Subject Descriptors

I.2.6 [**Artificial Intelligence**]: Learning—*Concept learning, Knowledge acquisition*; I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

General Terms

Algorithms, Design, Experimentation

Keywords

Co-evolution, Game theory, Belief, Incomplete Information

1. INTRODUCTION

Many social contacts are indirect. Often people obtain information from indirect sources. Based on information obtained, people may have unjustified or incorrect beliefs which do not correspond to reality. How beliefs change through interactions and how beliefs influence decision making are interesting questions. This paper attempts to understand beliefs from incomplete information in a simple game which has both direct and indirect interactions.

We choose to study the classic Rubinstein incomplete information alternating-offer bargaining problem [3]. Its structure of incomplete information is simple: only one player is not fully informed; his opponent has two possible types. Player i's time preference is determined by his *discount factors* δ_i. Player i knows his own δ_i. The first player, player 1 is uncertain about which is the player 2' real discount factor δ_2 and which is the incorrect δ_2' from two possible values: a smaller one δ_w of the weak type of player 2, 2_w and a larger δ_s of the strong 2, 2_s, where $\delta_2, \delta_2' \in \{\delta_w, \delta_s\}$, $\delta_s > \delta_w$, and $\delta_2 \neq \delta_2'$. Before the bargaining starts, player 1 believes that the possibility of $\delta_2 = \delta_w$ is ω_0. Rubinstein [3] solves this problem with *Perfect Bayesian Equilibrium* (PBE).

Copyright is held by the author/owner(s).
GECCO'06, July 8–12, 2006, Seattle, Washington, USA.
ACM 1-59593-186-4/06/0007.

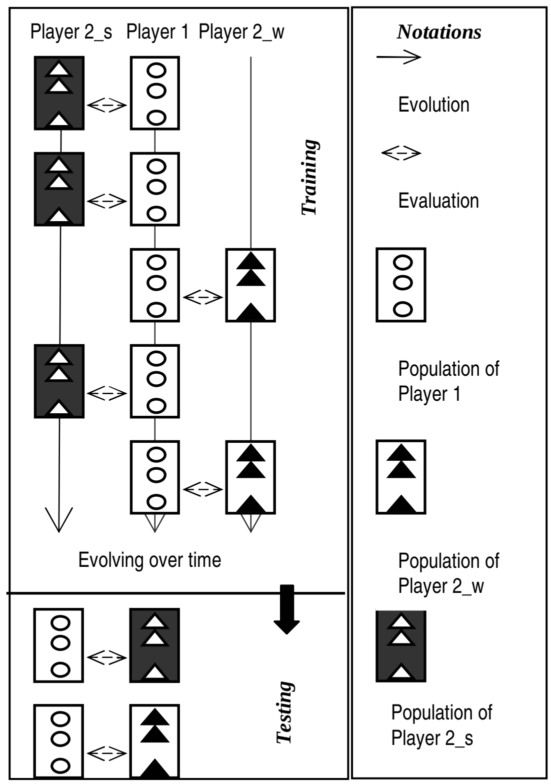

Figure 1: Mixture Co-evolution

We [2] started with a two-population co-evolution algorithm for this problem. Experimental results show that δ_1 and δ_2 almost entirely determine the bargaining outcomes in terms of their distribution, efficiency and stability. On the other hand, δ_2' and ω_0 in player 1' initial belief do not make any strategic difference on the outcomes. Such conclusions conflict with the game-theoretic analysis as well as observations from economics experiments.

In this paper, we will refine the co-evolution system to explicitly model and emphasize the three special features of this problem: (1) Two types of 2 are supposed to simultaneously affect player 1' decisions from theoretical and practical perspectives. The co-evolution system needs to train player 1 to learn how to bargaining with both types of player 2. (2) both 2_s and 2_w indirectly connect with each other through their common opponent player 1. (3) The game itself is defined as a one-off (or called 'single', 'one-slot') game. The system should separate the training and testing and evaluate the outcomes from playing the game once.

2. MIXTURE CO-EVOLUTION - TRAINING

The mixture co-evolution algorithm composes three interacting populations as illustrated in Figure 1. Note that this algorithm is not a simple three-population paired co-evolution [1]. The type of player 1 is unique, so he has one population. The other two populations for 2_s and 2_w, both co-evolve with player 1. Player 1 has two underlying objectives P_{2_s} and P_{2_w}.

Definition 1. Populations of Mixture co-evolution. A population P is a set of strategies. P_1 is the population of player 1. P_{2_s} and P_{2_w} are the population of player 2_s and of that of 2_w respectively.

Definition 2. Direct Co-evolution. If P_i and P_j interact in such way that individuals in one population are assessed by individuals in the other co-evolving population, P_i and P_j directly co-evolve. We denote $P_i - P_j$. The relative fitness function for P_i is $f_i(g_i, P_j(n))$, where the non-negative integer n is the evolutionary time.

Definition 3. State Transaction Equations of populations in the training period. Var is the variation operation in evolution (mutation and/or crossover). Sel is the selection operation. $Prob$ is the probability of the presence of $P_1 - P_{2_w}$ at n. Then the state transaction equation of Mixture Co-evolution is:

$$\begin{vmatrix} P_1(n+1) \\ P_{2_s}(n+1) \\ P_{2_w}(n+1) \end{vmatrix} = \begin{cases} \begin{vmatrix} Var(Sel(P_1(n), f_1(P_{2_w}(n)))) \\ P_{2_s}(n) \\ Var(Sel(P_{2_w}(n), f_{2_w}(P_1(n)))) \end{vmatrix} \\ \qquad \text{where } Prob = \omega_0; \\ \begin{vmatrix} Var(Sel(P_1(n), f_1(P_{2_s}(n)))) \\ Var(Sel(P_{2_s}(n), f_{2_s}(P_1(n)))) \\ P_{2_w}(n) \end{vmatrix} \\ \qquad \text{where } Prob = 1 - \omega_0. \end{cases} \tag{1}$$

Definition 4. Indirect Co-evolution. If $P_i - P_j$ and $P_i - P_k$, but $P_j - P_k$ is not true, then P_j and P_k indirectly co-evolve. We denote $P_j \sim P_k$. In the bargaining problem, player 2_w (or 2_s) is aware of the existence of 2_s (or 2_w) and of player 1's information structure. She expects that player 1's strategies take accounts into both types of 2's discount factors and the possibility of their appearance. Consequently, 2_s's strategies (or 2_w's) are in association with both 1 and 2_w (or 2_s).

The order of appearance of $P_1 - P_{2_s}$ and $P_1 - P_{2_w}$ is random, with the frequency of $P_1 - P_{2_s}$ being $(1 - \omega_0)$ of training examples and that of $P_1 - P_{2_w}$ being ω_0 of training examples. The emergences of 2_s and 2_w interweave.

[1]Ficici and Pollack [1] study a similar three-population co-evolving system. It has three critical differences from our work: (1) one population is solely determined by its own internal dynamics; in our work, all three populations are dynamically interacted; (2) The influence of the two predictors' populations on their common co-evolver, the generator population is of equal importance; in our work, which one of player 2's populations will encounter with the player 1's population is determined by the possibility ω_0 in player 1's belief, so the two populations for player 2 have, probably, imbalanced impact on the player 1's population;(3) [1]'s game is designed as repeated game; the game we attempt to study is an one-off game.

3. MIXTURE CO-EVOLUTION - TESTING

The trained player 1's population P_1 plays once with the trained player 2_w's population P_{2_w}. The same P_1 plays once with trained P_{2_s}. These two tests are executed without interference with each other. To separate testing from training helps ensure correct performance measure for this one-off game.

4. EXPERIMENTS

The mixture co-evolution system is implemented by Genetic Programming. Strategies are represented by syntax tree with the terminal set $\{\delta_1, \delta_w, \delta_s, \omega_0, 1, -1\}$ and the functional set $\{+, -, \times, \div\}$.

Game theory measure solutions on the properties of a bargaining outcome in terms of its *distribution*, *efficiency* and *stationarity* [3]. Distribution concerns how the mutual benefit is divided among participants. Efficiency measures whether an outcome is Pareto-efficient. Game -theoretic solutions are stationary as players have no desire to withdraw unilaterally from the equilibrium. We compare our experimental results with those [2] from the two-population co-evolution algorithm as well as with the game-theoretic solutions Perfect Bayesian Equilibrium. Experimental findings suggest that the information incompleteness adjust bargaining powers of preferences, partially cause inefficiency and delays, and decrease evolutionary stability. The application of the mixture co-evolution algorithm provides solutions that demonstrate the impact of player's belief and of information incompleteness on bargaining outcomes, consistent with game-theoretic explanations. Such results are considered to be realistic too. In practice, a player who has the advantage over his opponent on information about the game exploits his privilege of knowing the other's without disclosing his own information, particularly in one-off encounters.

5. CONCLUSION

In summary, the new algorithm delivers desired solutions from the viewpoints of game-theoretic analysis and experimental economics. Furthermore, this work infers that the effective artificial learning is unlikely achieved by merely adding information. Learning demands training experiences on how to deal with such information; Secondly, experimental observations suggest that after proper training, the similar impacts of indirect dependence on the outcomes appear as what game-theoretic solutions predict in Perfect Bayesian Equilibrium; Finally, especially for one-off games, it is essentially important to separate training and testing.

6. REFERENCES

[1] S. G. Ficici and J. B. Pollack. Challenges in coevolutionary learning. In *Proc. of the Sixth Int. Conf. on Artificial Life*, pages 238–247. The MIT Press, 1998.

[2] N. Jin. Equilibrium selection by co-evolution for bargaining problems under incomplete information about time preferences. In D. C. et al., editor, *Proceedings of the 2005 IEEE Congress on Evolutionary Computation*, 2005.

[3] A. Rubinstein. A bargaining model with incomplete information about time preferences. *Econometrica*, 53(5):1151–72, 1985.