Feasible Encodings For GA Solutions of Constrained Minimal Spanning Tree Problems

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Abstract

We solve by a genetic algorithm (GA), three NP-hard, constrained, minimal spanning tree (MST) problems on a complete graph using a novel encoding for the genotype which ensures feasibility of the search space when performing crossover and mutation and when initializing the population. By employing a feasible encoding the standard, mainstream, GA paradigm is preserved allowing us to capture the full benefits of the genetic operators. We do not need to resort to the elaborate and time consuming detection and repair operations proposed by other researchers to handle non feasible population members.

The feasible genotype for the degree constrained or leaf constrained MST problem is a permutation code which is suitably mapped into a Prufer code (phenotype) whose spanning tree obeys the desired tree constraints. This methodology can provide a framework for the development of feasible GA encodings for a wide class of other constrained minimal spanning tree problems on a complete graph.

The effectiveness and competitive performance of this methodology is demonstrated by convergence to a global optimum for various degree constrained MST problems including a benchmark problem often cited in the literature.

We investigate the Genetic Algorithm (GA) solutions of three NP-hard constrained MST problems on a weighted complete graph, viz, the order constrained MST problem with time dependent edge weights, the degree constrained minimal spanning tree (DCMST) problem and the leaf constrained minimal spanning tree (LCMST) problem . These important problems are of practical use in circuit and communications network design.

The unique strength of the paper is the development of innovative encodings for the genotype of the population members which ensures feasibility of the search space when performing the genetic operations of crossover and mutation and when generating the initial population .

By employing a feasible encoding, the standard, mainstream, GA paradigm is preserved allowing us to capture the full benefits of the genetic operators and to use , in a straightforward manner, any of the numerous enhancement techniques in the GA literature. We do not need to resort to the elaborate and time consuming detection and repair operations proposed by other researchers to handle non feasible population members. Our method avoids the loss of important genetic material inherent in repair mechanisms.

Building on our prior work with Prufer codes for feasible encodings of spanning trees [Edelson and Gargano, 1998] , we develop novel feasible GA encodings for the DCMST and LCMST problems. The genotype for each problem is a suitable permutation code (an adaptation of the simple idea of a permutation) which is feasible since permutation codes always give rise to feasible search spaces. Appropriate linear mappings are developed which transform the permutation code into a Prufer code (phenotype) whose spanning tree obeys the desired tree constraints. This methodology can provide a framework for the development of feasible GA encodings for a wide class of other constrained minimal spanning tree problems on a complete graph.

The GA methodology insures diversity of the population in that all Prufer codes (and their corresponding constrained spanning trees) are equally likely to be in the initial population and be selected as one of the parent pairs during mating. We demonstrate the effectiveness and competitive performance of this method by using a customized Visual C++ program to generate computational results for four DCMST problems including a benchmark problem often cited in the literature. Convergence to the global optimum is achievable with high mutation rates.

References

Edelson,W. , Gargano, M.L., Minimal edge-Ordered Spanning Trees Solved by a Genetic Algorithm With Feasible Search Space, Congressus Numerantium 135, (1998), 37-45.