Generating Robust Control Equations with Genetic Programming for Control of a Rolling Inverted Pendulum

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Abstract

In control applications, to apply the results obtained by evolutions and simulations to real systems, it is indispensable for them to have high robustness in computer simulations. The goal of this study is to generate robust control equations for controlling the real system of a rolling inverted pendulum with Genetic Programming. The control equations for this system must swing a pole up from a hanging state and then keep the pole inversely standing. Therefore, we introduce the "compound fitness evaluation," which consists of two simulations corresponding to two control requirements. As a result of the evolutionary experiments, five robust and stable control equations that swing a pole up and keep it standing are obtained. By theoretical analysis, it is proven that pole-cart systems using these control equations are asymptotically stable. The robustness of these equations are verified by dynamical simulations that vary each parameter of the pole-cart system.

1 INTRODUCTION

In control applications, to apply the results obtained by evolutionary methods and simulations to real systems, it is indispensable for them to have high robustness for variable situations in computer simulations. The goal of this study is to generate robust control equations for controlling the real system of a rolling inverted pendulum with Genetic Programming (GP).

The pole-balancing problem has previously been approached many times with methods such as evolutionary fuzzy logic (Karr, 1991) and evolutionary neural networks (Kaise, 1998). However, there are few approaches to the problem with equations generated by Genetic Programming (GP) (Shimooka, 1998). In particular, there have been almost no GP approaches with control equations for controlling a rolling inverted pendulum.

We have attempted this task with an evolutionary neural networks (ENNs) approach (Kaise, 1998). Neural networks that can swing a pole up and keep it standing have been achieved. However, these networks have unfeasible control patterns due to high frequency on-off control and have low robustness. Of course, it is possible to give ENNs control robustness by using the generalization characteristics of NNs, which requires tuning the number of hidden layer cells. Unfortunately, the ENN approach has provided few highly robust controllers.

In this study, we adopted a GP approach that directly evolves robust control equations for the rolling inverted pendulum used for calculating the driving force of a pole cart system. The reason why we adopted the GP approach is that it is expected that control equations simplified by the parsimony factor and the depth limitation of trees in the GP process have generalization mechanisms themselves and are thus robust for control. Moreover, a key point of the GP approach is that the expression of a control equation facilitates theoretical analysis of control systems. In this paper, the stability of the pole-cart system with the control equations obtained in the experiments is analyzed theoretically.

The control equations must swing a pole up from a hanging state and then move a cart to a given target position while keeping the pole standing. Therefore, we introduced the "compound fitness evaluation," which consists of evaluations with two different simulations. One is a simulation that starts with the pole in the hanging state. The other is a simulation that

starts with the pole in a roughly standing state.

The robustness of the control equations obtained in the experiments on GP evolution are tested by theoretical analysis and dynamical simulations of the pole-cart systems, in which the parameters are varied in the range of ±50%.

2 ROLLING INVERTED PENDULUM

2.1 THE MODEL OF A ROLLING INVERTED PENDULUM

A rolling inverted pendulum task is a control problem of a pole-cart system; one end of the pole is jointed to a rotary shaft on the cart. The purpose of the task is to swing the pole up from the hanging state and then stabilize the pole in an inversely standing position.

The pole-cart system is simulated in two-dimensional space, i.e. the state vector $\mathbf{x} = (\dot{\theta}, \dot{x}, \theta, x)$, where $\dot{\theta}, \dot{x}, \theta$ and x are the angular velocity, the cart velocity, the pole angle and cart position, respectively. The pole angle is defined as 0 rad when the pole is standing upright on the cart. In addition, no friction of the rotary shaft or sliding of the cart is assumed.

The equations of motion given by (Anderson, 1986) is simulated in discrete times with the Runge-Kutta and the Euler approximation methods for a differential equation. For these simulations, the constants are the time step ($\Delta t = 0.02$ seconds), the mass of the cart ($m_c = 1.0$ kg), the mass of the pole ($m_p = 0.1$ kg), the pole length ($l = 1.0$ m), and gravity ($g = 9.8$ m/s^2).

2.2 STABILITY OF THE POLE-CART SYSTEM

The state equation of the pole-cart system is given by Eqs. (1) and (2):

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, u) := \begin{pmatrix} f_1(\mathbf{x}(t), u) \\ f_2(\mathbf{x}(t), u) \\ \dot{\theta}(t) \\ \dot{x}(t) \end{pmatrix}, \quad (1)$$

$$u = force(\mathbf{x}). \quad (2)$$

The paper of Anderson (Anderson, 1986) is referred to for $f_1(\mathbf{x}, u)$ and $f_2(\mathbf{x}, u)$. Equation (2) is the control equation generated by GP evolution. Substituting Eq. (2) into Eq. (1), Eq. (3) is derived.

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, force(\mathbf{x})) =: \mathbf{F}(\mathbf{x}). \quad (3)$$

The vector $\mathbf{x}_0$ of the target state is

$$\mathbf{x}_0 = {}^{\mathrm{t}}(0, 0, 0, T).$$

Note that if

$$\left. \frac{d\mathbf{x}}{dt} \right|_{\mathbf{x}_0} = \mathbf{F}(\mathbf{x}_0) = \mathbf{0} \quad (4)$$

is satisfied, $\mathbf{x}_0$ is the equilibrium point of the dynamics given by Eq. (3). The condition for satisfying Eq. (4) is

$$u = force(\mathbf{x}_0) = 0. \quad (5)$$

Therefore, we examine whether the target state $\mathbf{x}_0$ is the equilibrium point or not by verifying whether the condition Eq. (5) is satisfied or not.

The following equation is derived by linearizing Eq. (3) around the equilibrium point $\mathbf{x}_0$:

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}, \quad (6)$$

where the (i, j) component of $\mathbf{A}$ is

$$A_{i,j} = \frac{\partial \mathbf{F}_i}{\partial \mathbf{x}_j}(\mathbf{x}_0), \quad (7)$$

and $\mathbf{F}_i$ is ith element of $\mathbf{F}$, $\mathbf{x}_j$ is jth element of $\mathbf{x}$. In order to examine the stability of the pole-cart system with a control equation obtained by experiments, eigenvalues of $\mathbf{A}$ are calculated. If the real part of eigenvalues are all negative values, the system is asymptotically stable around the equilibrium point $\mathbf{x}_0$. If there is at least one zero value among nonpositive values in the real part of eigenvalues, the system may sustain oscillation around $\mathbf{x}_0$. Also if there is at least one positive value in the real part of eigenvalues, the system is unstable.

3 APPLYING GP TO THE ROLLING INVERTED PENDULUM

3.1 FUNCTION SET AND TERMINAL SET

In this study, the force needed to control the cart is directly expressed as an equation defined by a tree of S-expression with a function set and a terminal set in GP. For this problem, the function set and the terminal set are prepared as follows:

$$\mathcal{F} = \{+, -, *, \%, \sin, \cos\},$$
$$\mathcal{T} = \{\dot{\theta}, \dot{x}, \theta, d, 1.0, 10.0, -1.0, 2.0, 3.0, 4.0, \pi\},$$

where % is the modified division defined by Koza (Koza, 1992). The function set is the most elemental set of the four arithmetic functions and two periodic functions. The terminal set includes the parameters of the pole-cart system: $\dot{\theta}$, $\dot{x}$ and θ. Moreover, it includes d, which is the difference between the cart and the target position.

3.2 COMPOUND EVALUATION

The objective of this study is to search for robust control equations that will allow the pole to swing up from the hanging state and become stabilized while standing, and then allow the cart to move to a given target position. We divided the requirements of the objective into two parts, i.e. the motion of swinging the pole up and the motion of moving the cart to the given target position. That is, two simulations are conducted to evaluate one control equation. The first simulation starts in the pole hanging state. The second simulation starts with the pole in a roughly standing state. That's because no desired control equation can be obtained by using an evaluation with a simple simulation that starts with the pole in a hanging state.

In this study, the evaluation function is defined as a minimum search problem. When $fitness^{(1)}$ is defined as the fitness by simulations that start the pole in a hanging state and $fitness^{(2)}$ is defined as the fitness by simulations that start with the pole roughly standing, then the $fitness$ of a control equation can be calculated by equations as follows:

$$fitness \;=\; \sum_{i=1}^{2} \omega^{(i)}\, \frac{fitness^{(i)}}{max^{(i)}}, \qquad (8)$$

$$fitness^{(i)} \;=\; \sum_{t=1}^{t_i - 1} \Big[\omega_1^{(i)}\, g\big(\theta(t)\big) + \omega_2^{(i)} |x(t) - T| \Big]$$
$$+\; \omega_3^{(i)} \times s^{(i)} \;+\; \sum_{t=t_i}^{STEP} \omega_4^{(i)}\, x_{max}^{(i)}, \qquad (9)$$

$$s^{(i)} = \begin{cases} t_s^{(i)} & ([\,\dot{\theta}^2(t_s^{(i)}) + \theta^2(t_s^{(i)}) \\ & \qquad\qquad + \dot{x}^2(t_s^{(i)})\,] < \varepsilon \\ & \text{and } |x(t_s^{(i)}) - T| \le \delta), \\ STEP & \text{(otherwise)}, \end{cases}$$

$$max^{(i)} \;=\; \omega_3^{(i)}\, STEP + \omega_4^{(i)}\, x_{max}^{(i)}\, STEP,$$

$$g(\theta) = \begin{cases} |\,[\,(\theta - \pi)\ (\mathrm{mod}\ 2\pi)\,] - \pi| & (\theta \ge \pi), \\ |\,[\,(\theta - \pi)\ (\mathrm{mod}\ 2\pi)\,] + \pi| & (\theta < \pi), \end{cases}$$

where t_1 is the time when the condition $|x(t_1)| > x_{max}^{(1)}$ is satisfied, and t_2 is the time when the condition $|x(t_2)| > x_{max}^{(2)}$ or $|\theta(t_2)| > \theta_{max}^{(2)}$ is satisfied. $g(x)$ is the periodic function of the pole angle. ε is the small

constant to decide the stationary state of the pole, and δ is the allowance for the error between the cart and the target position; $\varepsilon = 10^{-6}$, $\delta = 10^{-3}$. Parameters in the evaluation function are given in Table 1.

Table 1: Parameters in Evaluation Function

Parameter	Value	Parameter	Value
$\omega^{(1)}$	0.05	$\omega^{(2)}$	1.0
$\omega_1^{(1)}$	50.0	$\omega_1^{(2)}$	0.1
$\omega_2^{(1)}$	0.1	$\omega_2^{(2)}$	10.0
$\omega_3^{(1)}$	0.1	$\omega_3^{(2)}$	0.1
$\omega_4^{(1)}$	50.0	$\omega_4^{(2)}$	50.0
$x_{max}^{(1)}$	5.0	$x_{max}^{(2)}$	15.0
		$\theta_{max}^{(2)}$	$\pi/4$

4 EMPIRICAL PROCEDURE

In the experiments, the SGPC 1.1 program developed by W.A. Tackett and A. Carmi is used for GP simulations. The important parameters of GP are set at population size = 3000, maximum generation = 100, and parsimony factor = 0.00001. The paper of Kinner (Kinner, 1993) is referred to for the parsimony factor. The depths limit of trees are set at $D_{initial} = 8$, $D_{created} = 20$, and $D_{mutant} = 6$.

In the evaluations, the initial states and the target position in the simulation starting in the hanging state are as follows: $\dot{\theta}(0) = 0.0$, $\theta(0) = 0.0$, $\dot{x}(0) = 0.0$, $x(0) = 0.0$, $T = 0.0$, and the initial states and the target position in the simulation starting in the roughly standing state are as follows: $\dot{\theta}(0) = 0.0$, $\dot{x}(0) = 0.0$, $\theta(0) = [\pi/3 + \mathrm{Rnd}(-\pi/36,\ \pi/36)]\ (\mathrm{mod}\ \pi/3) - \pi/6$, $x(0) = \mathrm{sgn}(\mathrm{Rnd}(-1.0,\ 1.0)) \times 10.0$, $T = 0.0$. $\mathrm{Rnd}(a, b)$ means the generation function of random numbers between a and b, and $\mathrm{sgn}(x)$ means the sign function; if $x \ge 0$, then $\mathrm{sgn}(x) = +1$ else $\mathrm{sgn}(x) = -1$. A simulation is carried out for 1500 steps, i.e. 30 seconds.

5 EMPIRICAL RESULTS

5.1 PERFORMANCE OF CONTROL EQUATIONS

In this section, we show the performance results of control equations obtained as solutions of GP evolutions in 30 experiments. Simulations with the pole hanging in initial state are carried out using control equations obtained from the 30 experiments. As a result, there

are twelve control equations out of thirty equations that can allow the pole to swing up and then keep the pole standing. However, not all of the equations can stabilize the pole in the standing state at a target position.

Next, we theoretically investigate the stability of the pole-cart system with each of the twelve control equations that can allow the pole to swing up. We examined whether the pole standing state on the cart at a target position, i.e. the state $\mathbf{x}_0 = (0, 0, 0, T)$, is the equilibrium point or not by means of substituting the state $\mathbf{x}_0$ into the twelve control equations. As a result, there are six among twelve control equations in which $\mathbf{x}_0$ is the equilibrium point.

For these six equations, we examined the stability at the equilibrium point by means of linearizing the equations of the dynamical system around the equilibrium point $\mathbf{x}_0$. The results for the stability of the six control equations are shown in Table 2. The five equations

Table 2: Stability of Six Control Equations

Stability	Rate
Asymptotically Stable	5 / 6
Sustained Oscillation	0 / 6
Unstable	0 / 6
Analysis Impossible	1 / 6

are asymptotically stable. There are no equations that may sustain the oscillation around the state $\mathbf{x}_0$. There are no unstable equations. One equation cannot be theoretically analyzed because it has a zero division that is defined as 1 in the function set of GP.

In these experiments, five asymptotically stable equations and one equation with zero division are contained in the robust equations experimentally defined in the previous paper (Shimooka, 1998). That is, it appears that the robustness experimentally defined in the previous paper involves the theoretical conditions of stability for control of the rolling inverted pendulum. Furthermore, the five equations are the only control equations that we intend to generate.

5.2 HOW TO CONTROL

The following Eq. (10) and Eq. (11) are simplified control equations from equation trees with the highest and the next highest robustness in thirty control equations:

$$force_1(\dot{\theta}, \dot{x}, \theta, d)$$
$$= 5\,\dot{\theta} + \sin(\dot{\theta}) + \dot{x} + \theta\,(\theta - 3.03)$$
$$+ 25.15\sin(\theta) - 0.25\sin(\theta\,d) + 0.33\,d, \quad (10)$$

$$force_2(\dot{\theta}, \dot{x}, \theta, d)$$
$$= \dot{x} + 9.09\,\theta + \left(6.0\,\dot{\theta} + 19.1\,\theta + d\right)\Bigg[\cos(\theta)$$
$$- \sin\left(\cos\left(\frac{d\,(\cos(\theta) - 0.02)}{\cos(2\cos(\theta))}\right)\right)\Bigg]. \quad (11)$$

The control processes by control Eq. (10) are shown in Figures 1 and 2, and the control processes by control Eq. (11) are shown in Figures 3 and 4. The right figure in Figure 4 is a magnified view of the left one. The simulations are carried out with two initial values of the pole angle:

$$\text{(i)} \quad \theta(0) = \pi, \qquad \text{(ii)} \quad \theta(0) = 7\pi\,/\,36.$$

Other parameters are set at 0, and the target position is also set at 0. Figures 1 and 2 shows that the control
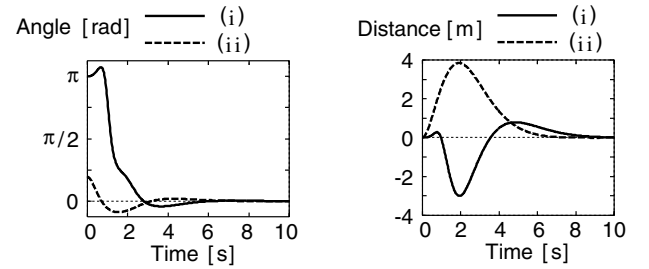

Figure 1: Transitions of Angle and Distance between Cart Position and Target Position for Conditions (i) and (ii) by Control Eq. (10)
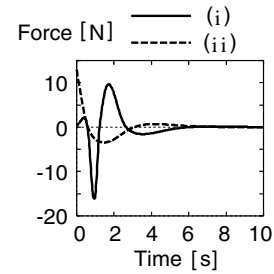

Figure 2: Transition of Force added to the Cart for Conditions (i) and (ii) by Control Eq. (10)

equation makes the pole swing up after one swing then stabilizes the pole-cart at the target position.

Figures 3 and 4 show that the control equation makes the pole rise up without any swings. However, the magnitude of the driving force is too large in the first few steps when the initial value of the angle is π. For implementation in real applications, the magnitude of the force must be limited.
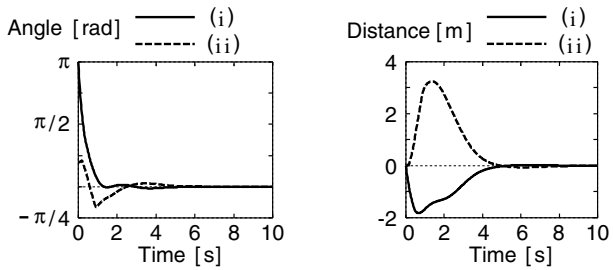
Figure 3: Transitions of Angle and Distance between Cart Position and Target Position for Conditions (i) and (ii) by Control Eq. (11)
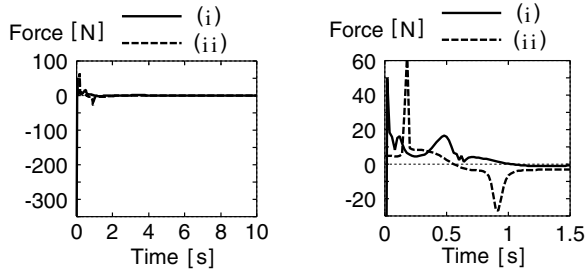

Figure 4: Transitions of Force added to the Cart for Conditions (i) and (ii) by Control Eq. (11)

5.2.1 STABILITY ROBUSTNESS

We now examine the stability robustness of control Eqs. (10) and (11). Simulations with the pole hanging in the initial state are carried out using each of the control equations while changing each parameter of the pole-cart system, that is, m_c, m_p and l. Each of these parameters is changed in the range of $\pm 50\%$. Table 3 shows the maximum and the minimum percentages of the pole-cart parameters where each control equation can swing a pole up and stabilize the pole in the standing state at a target position. Table 3 shows

Table 3: The Results of Stability Robustness Simulations Using the Control Eqs. (10) and (11)

	Control Eq. (10)			Control Eq. (11)		
Parameters	m_c	m_p	l	m_c	m_p	l
Max (%)	101	125	103	120	150	140
Min (%)	94	50	90	50	50	92

that Eqs. (10) and (11) have stability that is robust enough for application to the real world.

6   CONCLUSIONS

In this study, the GP approaches to the rolling inverted pendulum problem are attempted, and six ro-

bust control equations, which can swing a pole up and then move a cart to a given target position while keeping the pole inversely standing, were obtained by using evolutions with "compound evaluation." From the theoretical analysis, five control equations were validated to be asymptotically stable at the equilibrium state for the pole-cart system. Furthermore the robustness of the pole-cart systems with the two best control equations were also verified by dynamic simulations in which each parameter of the pole-cart system was varied in the range of $\pm 50\%$.

Finally, it may be expected that these control equations will be a hint for researchers in the field of control and contribute to the progress of control theories.

7   FUTURE WORKS

In future works, we will reduce the maximum force to a feasible magnitude and improve the robustness of the pole-cart systems by using these control equations in computers and applying them to a real pole-cart system of the rolling inverted pendulum.

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