A GENETIC ALGORITHM FOR THE MINI-MAX SPANNING FOREST PROBLEM

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Consider an election of r seats in country consisting of r cities and several villages. The problem is to divide the country into r electoral districts in such a way that each district consists of a city and some villages which are not separated by other districts. The total population of each district must be as balanced as possible. Generally, as each district can be regarded as the nodes on a spanning tree with the root node of a city, we may formulate above problems as a spanning forest problem. Evidently, a spanning forest consists of a set of mutually disjoint spanning trees. Given a set of root nodes $U := \{u_1, u_2, \ldots, u_r\}$ in a undirected graph $G = (V, E)$, an U-rooted spanning forest F is a spanning forest of G consisting of r disjoint trees $T_1, T_2, \ldots, T_r$ such that u_i is a node of T_i ($i = 1,2,\ldots,r$). As each edge $e \in E$ has an associated integer weight $w(e) > 0$, the weight $w(T)$ of a tree T can be defined as the sum of the weights of its constituent edges. Therefore, the mini-max formulation of the spanning forest problem can be formulated as follows:

$$w(F) = \min \max_{1 \le i \le r} \{ w(T_i) \}$$

This formulation of the spanning forest problem is denoted as the mini-max spanning forest problem (MMS-FP for short). We wish to find the U-rooted spanning forest F^* that minimizes $w(F)$ over all U-rooted spanning forests of G.

Clearly, for $r = 1$, the problem is the well-known minimum spanning tree problem, which can be solved by the polynomial time algorihtms such as Prm's[1] and Kruskal's. For $r \geq 2$, however, it has been proved as NP-hard. Only a heuristic algorithm was proposed by Yamada et al. to deal with the MMSFP. Obviously, there is much possibility to improve the algorithm to solve this problem. In this paper, we develop a genetic algorithms (GAs) approach to solve this problem As the problem solution takes a spanning tree structure with a dummy rooted nodes as shown in Figure 1, the GAs approach on this problem takes the tree-based genetic representation as shown in Figure 2, and order crossover, exchange mutation rate and insertion mutation are adopted for its genetic operations.

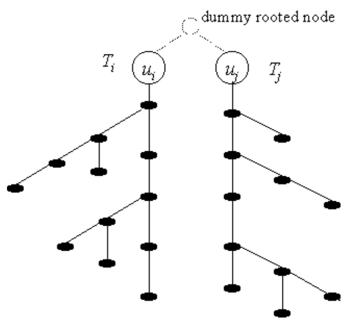

Figure 1: Illustration of a spanning tree solution

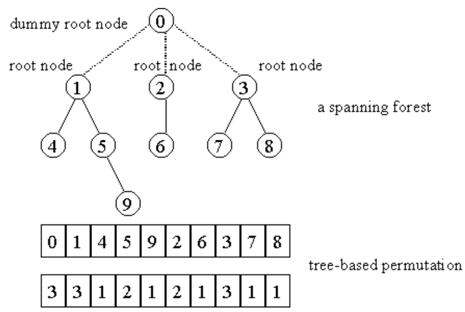

Figure 2: Illustration of its genetic representation

The numerical examples of the MMS-FP were designed as follows: the cost matrix was taken as the integer uniformly and randomly distributed in the range of (0, 80]. By using the genetic algorithm, the parameters were set as follows: population-size = 200; max-generation = 1000; order crossover rate = 0.4; exchange mutation rate = 0.3; and insertion mutation rate = 0.8. Numerical examples show that the high effectiveness of the proposed GAs approach can find much better solution than the existing heuristic algorithms and even the optimal solution or near-optimal solution with great probability., though it takes more CPU time to get the near-optimal solutions. However, it is acceptable in practical network design.