A Genetic Algorithm with Tabu Search for Multimodal and Multiobjective Function Optimization

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Abstract

The integration of genetic algorithms (GAs) and tabu search is one of traditional problems in function optimization in the GA literature. However, most proposed methods have utilized genetic algorithms to explore global candidates and tabu search to exploit local optimal points. Unlike such methods so far, this paper proposes a new algorithm to directly store individuals into multiple tabu lists during GA-iterations. The tabu lists inhibit similar solution candidates from being selected so often. The proposed algorithm is so simple but strong that we can solve both multimodal and multiobjective problems in the same manner. The paper describes the basic idea, algorithms, and experimental results.

1 INTRODUCTION

Hybrid genetic algorithms or the integration of genetic algorithms, simulated annealing, tabu search, and/or heuristics have been studied for long years to let GAs more powerful to solve complex optimization problems. Most of the conventional methods utilize GAs to explore global candidates and the other additional algorithms to exploit local optimal points. Unlike such conventional methods so far, this paper proposes new algorithms to directly store individuals into multiple tabu lists.

The Tabu lists have roles of (i) storing superior individuals in the previous generations, (ii) reusing the individuals as the elite (iii) maintaining diversity of the population, and (iv) inhibiting individual from converging local minima as is found in conventional Tabu search methods. Therefore, hence the optimization proceeds within the dynamical changes of the solution landscape, the Tabu-GA will be easier, more robust, and more powerful than the conventional hybrid methods.

The objectives of our research are to develop new GA-based methods, which enable us to simultaneously acquire multiple feasible solutions for both problems, multimodal and multiobjective function optimization.

The basic idea of the algorithms is to store the best solutions of each generation into long-term and short-term tabu lists, which inhibit the individuals from being selected more than n times. The tabu lists or tabu constraints depress the possibility to local convergence in the early stages of the iterations. This enables the candidates to explore new solution spaces to get better and/or various solutions. The final results are accumulated in the long-term tabu list. This means that multiple peaks are obtained for multimodal problems and that Pareto optimal solutions are obtained for multiobjective problems.

When applying the methods to multimodal problems, in order not to converge into one peak, we first measure Hamming distances between the individuals of the current generation and the ones in the tabu lists, then omit the individuals within the distance *d*.

When applying the methods to multiobjective problems, in order to acquire the better Pareto optima, we prepare multi-class tabu lists, each of which contains solutions of each objective function.

This paper is organized as follows. In section 2, we give the problem formulation and proposed algorithms. In section 3, to validate the effectiveness of the proposed methods, we carry out intensive experiments; the objective functions include Rastrigin, FMS-parameter, and multiobjective function. In section 4, we survey related work on hybrid methods in genetic algorithms. Finally, in section 5, concluding remarks and future work are described.

2 GENETIC ALGORITHM WITH MULTIPLE TABU-LISTS

This section describes the genetic algorithm with multiple tabu-lists, which aims at implementing a fast, simple, and robust method to get optimal points for both multimodal and multi-objective problems. The algorithm is unique because we can process both problems within the same framework and without explicitly considering the existence of schematic structures of the problem representation. The main idea of the algorithm is that, (1) in each generation, one best individual generated by GA operation is stored into the tabu-lists to inhibit it from selecting specified times, and (2) solution candidates found in the previous generations will become tabus, and thus, the other candidates are explored in order to get better and divergent solutions.

2.1 STRUCTURES OF THE TABU-LISTS

We have two kinds of tabu-lists in general: long-term list with the length *m* and short-term list with the length *n*. The *m* and *n* are parameters of the algorithm and can be tuned against given problems. The tabu-lists have the following four roles: i) storing superior individuals in the previous generations, ii) reusing the individuals as the elite, iii) maintaining the diversity of the population, and iv) inhibiting individuals from converging local minima. When solving multi-objective problems, the tabu-lists are extended to a multi-class so that each set of long-term and short-term tabu-lists is corresponds with (1) each objective function and (2) one Pareto optima. Furthermore, in the following sections, members of the long-term tabu list will be modified so that it only includes schematic information of the individuals. Using these tabu lists, we can simultaneously approach to multimodal and multiobjective problems.

Long-term Tabu List

It contains best *m* individuals during all previous iterations. The individuals in the long-term tabu list do not have the same or similar genotype.

Short-term Tabu List

It contains best *n* individuals during recent *n* iterations. The individuals in the short-term tabu list may have the same genotypes. The individuals only remain the *n* iteration, then are replaced in FIFO manner.

Multi-class Tabu List

When applying GAs to multiobjective problems, they reports that the optimization processes for one objective functions will be of use for generating the better Pareto optima. The multi-class tabu lists are prepared to correspond with each objective function. We also prepare another tabu list, which corresponds with the Pareto optima. The structures of the tabu lists are the same with the above long- and short-term tabu lists.

2.2 ALGORITHM OF TABU-GA FOR A MULTIMODAL FUNCTION PROBLEM

After evaluating each individual by means of the objective function in each iteration, we store the best individual of the generation into both long-term and short-term tabu lists. When selecting parents‘ candidate by the tournament selection method, we refer to the tabu lists in order not to select individuals with similar genotypes by means of the Hamming distance. The tabu constraint can be applied to only one parent to generate offspring. Using the tabu constraints, we also avoid converging the individuals to local optima. The solutions are gradually accumulated into the long-term tabu list. Thus, in case of a multimodal function, (respectively, a multiobjective function), multiple solutions (respectively, Pareto solutions) are obtained, simultaneously. The outline of the algorithm is shown in Figure 1.

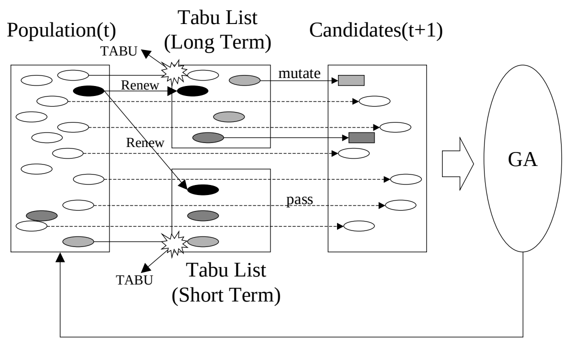

Figure 1: Tabu-GA.

1. Set H Empty, H which is a historical memory. Select $x^{now} \in X$ as initial solutions.
2. Select $N(H, x^{now})$, where $N(H, x^{now})$ is a set except the neighborhoods of H in x^{now} .
3. Select $x^{next} = fitness\_selection(c(N(H, x^{now})))$, where $c(\ )$ is a value set with an objective function, *fitness_selection* is a GA Selection method.
4. Run $x^{next} = crossover(x^{next})$ and $mutation(x^{next})$.
5. Evaluate $x^{best} = best\_fitness(x^{next})$.
6. If a condition of ending is true then end.
7. If x^{best} is better than x^{old} in H, and not similar to x^{old} in H, exchange x^{old} for x^{best}.
8. Add $mutation(x^{best})$ to X.
9. Return.

3 UPDATING THE TABU LISTS TO KEEP THE DIVERSITY

To avoid the solutions to converge to one peak for a multimodal function, we extend the tabu constraint, where the distance of an individual in the tabu list and a new candidate is less than d. We employ the following distance measures.

3.1 MEASURING DIVERSITY

Hamming distance

It represents the difference of bits in the two genotypes.

$$d_H(\mathbf{a},\mathbf{b}) = \sum_{i=1}^{n} |\mathbf{a_i} - \mathbf{b_i}|$$

Schema matching

It represents the similarity of schemas contained in the two genotypes.

$$d_S(\mathbf{a},\mathbf{b}) = \sum_{i=1}^{n} |\,schema(\mathbf{a})_\mathbf{i} - schema(\mathbf{b})_\mathbf{i}\,|$$

Norm

It represents the difference of the values of phenotypes, or function values of the two individuals.

$$d_N(\mathbf{a},\mathbf{b}) = \sum_{i=1}^{n} \|\,ptype(\mathbf{a}) - ptype(\mathbf{b})\,\|$$

3.2 MANAGING TABU LIST

We implement the following tabu list updating methods in order to keep the diversity of the individuals. Although the parameters depend on the characteristics of the problem domain, we could not find the remarkable difference among them from our intensive experiments. One point we should mention is that, in case of the schema tabu, the threshold value to detect the schemas is very sensitive to get good solutions.

Genotype tabu

When selecting individuals generated by specific GA operations via the tournament selection method, compare them with the ones in the tabu list. If the fitness value of the selected individual is low and the distance between the genotype of the selected individual and the genotype in the tabu list is within d_H by means of Hamming distance, then the one in the tabu list remains.

Schema tabu

If the same schema is kept for a long duration in the iteration, it might be a part of the candidate of global and/or local optima. We detect the schemas by (1) representing the current individuals to a matrix form and (2) finding the convergence of the phenotypes by measuring each locus with a given threshold value. If the fitness of the best individual of the generation is low and the distance between the detected schema and the schema in the tabu list is within d_S, then the schema in the tabu list remains.

Phenotype tabu

The method is the same with the above one, except the distance d_N of the detected schema and the schema in the tabu list is evaluated by the phenotype of the individuals.

3.3 ALGORITHM OF TABU-GA FOR MULTIOBJECTIVE FUNCTIONS PROBLEM

When applying our algorithm to multiobjective problems, we prepare multi-class tabu lists: the ones for each objective function and a tabu list for Pareto optima. Thus, the number of the tabu lists is m+1, where m is the number of the objective Functions. The Pareto optima are evaluated by the ranking method. Each offspring is evaluated by each tabu list. The individuals are selected by the tournament selection by means of each objectives and Pareto optima. The short-term tabu list is replaced with FIFO method, and the long-term tabu list is replaced with the rank of the tabus and the latest best fitness individual. Genetic operations are applied to all the individuals. The outline of the algorithm for multiobjective functions is shown in Figure 2.

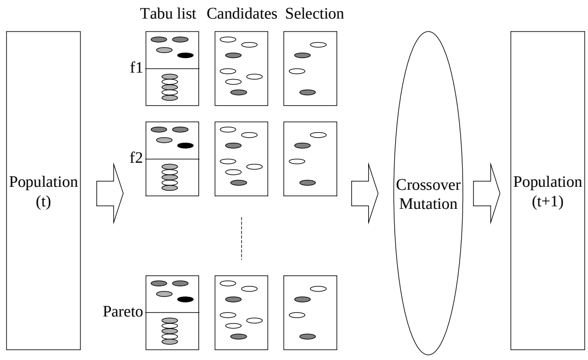

Figure 2: Multiobjective Tabu-GA.

4 EXPERIMENTS

To validate the effectiveness of the proposed method, we carry out numerical experiments on various test functions, which include very difficult multimodal functions.

4.1 EXPERIMENTAL SET UP AND OBJECTIVE FUNCTIONS

We compare the proposed method with Simple GA with the tournament selection method. Simple GA is also modified so that it processes both multimodal and multiobjective functions. We employ the following functions as the test bed.

- Sin function

$$\max \quad : \quad f_{\sin}(x) = \frac{a_1 \sin(2\pi x + a_2)}{a_3(x + a_4)}$$
$$a_i : const.$$

- Rastrigin function

$$\min \quad : \quad f_{ras}(x) = nA + \sum_{i=1}^{n} x_i^2 - A\cos(2\pi x_i)$$
$$i = 1,2,3,\ldots,n \quad A = 10.$$

- FM sound parameters function

$$\max \quad : \quad f_{fms,i}(x, y) = x_i \sin(2\pi y_i t + f_{fms,i+1})$$
$$i = 1,2,3,\ldots,6.$$

- Multiobjective function

$$\min \quad : \quad f_1(x) = \frac{x_1^2}{4}$$
$$\min \quad : \quad f_2(x) = x_1(1 - x_2) + 5$$
$$s.t. \qquad 1 \le x_1 \le 4, \quad 1 \le x_2 \le 2.$$

Table 1: summarizes the experiment parameters.

Function	Parameters
Sin	TGA(MM,20,3,1,H 0.9)
Rastrigin	TGA(MM,50,10,3,H 0.9)
FMS-parameters	TGA(MM,50,10,3,N 0.2)
Multiobjective	TGA(MO,50,5,3,N 0.2)

Where TGA(MM, i, m, n, d) shows the characteristics of Tabu-GA, MM/MO shows multimodal or multiobjective, i is the number of individuals, m is the length of long tabu lists, n is the length of short tabu lists, and d is the distance of individuals: H/Hamming; N/Norm. Common methods of them are Tournament Selection and Uniform Crossover.

4.2 EXPERIMENTS IN TABU-GA

To each test function, we have applied both Simple GA and Tabu-GA. General observations have suggested that (1) Individuals generated by simple GA with conventional elitist strategies loose their population diversity, and then rapidly converge to local optima, (2) On the other hand, the tabu-GA has the more wider searching area, and escapes the individuals from local optima, thus, it finds global optima more often.

4.3 SIN FUNCTION

The cases of the sin function are summarized in Figure 3. The proposed method works better than Simple GA with the sharing method, that is, many peaks are simultaneously obtained. The remarkable point is that the change of the number of the tabu list size easily controls the number of multiple solutions compared with the sharing method. The effect of the tabu list size is shown in Table 2. The tabu size of the upper figure is three, and the lower figure is ten. Both figures certainly show that tabu lists can get the diversity depending on the size. Where the population size is 20, the mutation rate is 0.005. We have also compared the results with the ones employed a sharing function in (Goldberg,1989), which have revealed that the parameters of the sharing functions are very sensitive to the number of solutions obtained.

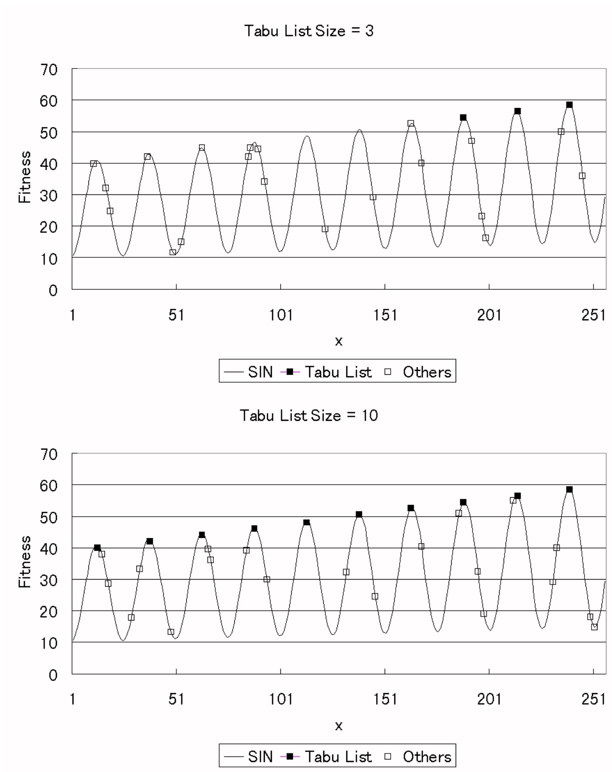

Figure 3: Sin function.

4.4 RASTRIGIN FUNCTION

Figure 4 shows the landscape of Rastrigin function, which is converted to the maximization problem. Simple GA with or without hill climbing techniques cannot generate optima of such functions with many peaks. The proposed method finds the optima faster than simple GA. The result is shown in Figure 4.

Table 2: Effect of Tabu List's Size on SIN Function.

Tabu List Size	The Number of Generation to Converge	The Number of Multimodal Solutions
3	36	3
7	95	7
10	187	10

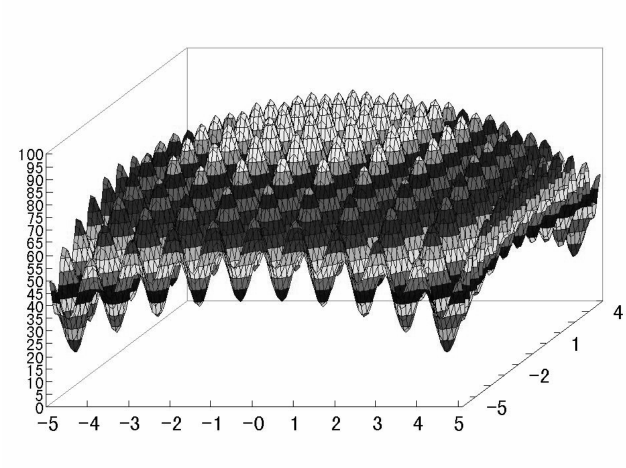

Figure 4: Rastrigin function (f, x_1, x_2).

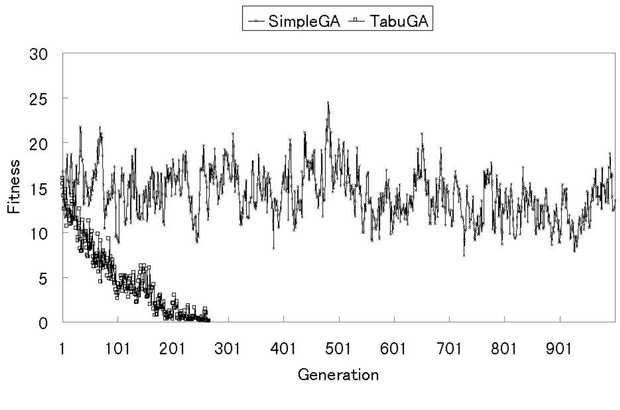

Figure:5 Tabu-GA results of Rastrigin function.

4.5 FMS PARAMETER FUNCTION

FMS parameter function was introduced by (TsuTsui93) to show the effectiveness of their folking GA method. The landscape is shown in Figure 6 with parameters n=6, and y_2-y_3, which is also converted to the maximization problem. As shown in the figure, searching optima in the y_2-y_3 space is a complex multimodal problem.

Figure 7 shows the iteration processes by Simple GA, Simple GA with the elitist strategy, and the tabu-GA. Ten cases of Simple GA and the tabu-GA are all averaged in the figure. Simple GA fails to find the optimal solution after 1,000 iterations with 50 individuals, Simple GA with the elitist strategy finds the optimal solution two times from the ten trials. About the other eight cases, the solution rapidly converges to local optima. The figure plots the results of the three cases out of the eight trials. On the other hand, the proposed method always finds the optima in the ten trials.

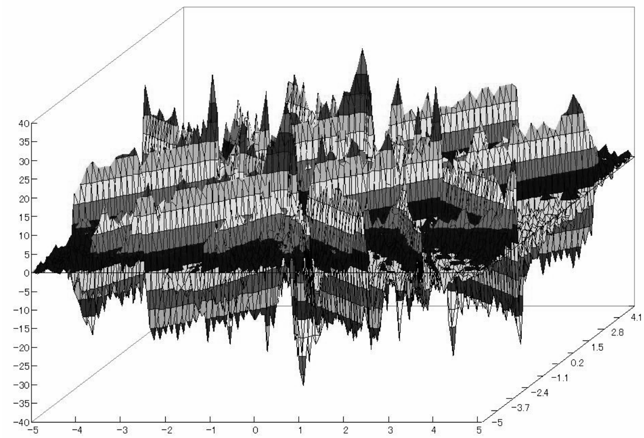

Figure:6 FMS function(f, y_2,y_3).

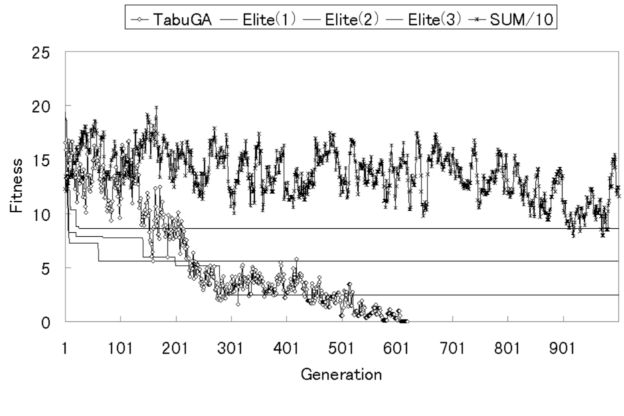

Figure:7 Tabu-GA results of FMS function.

Figure 8 shows the final stage of the iteration. Similar to the sin function case, we get three local optima as marked A, B, and C in the figure. Among the three C is the global minimum. Please note that the tabu list contains these local optima, which are hardly obtained by conventional GAs.

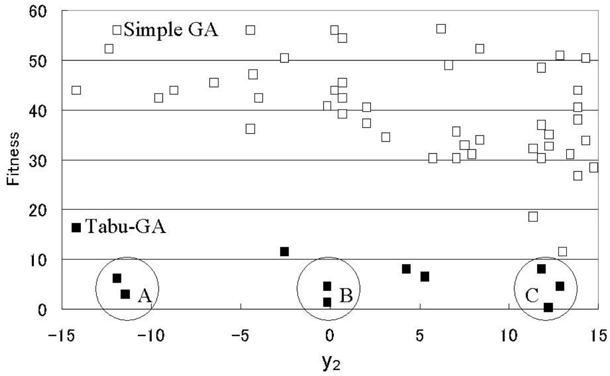

Figure :8 Individuals of FMS function.

4.6 OPTIMIZATION OF A MULTIOBJECTIVE FUNCTION

Conventional GAs including the simple GA only process single objective functions or multiobjective functions which are represented by the linear combination of each Function.

Contrary, the proposed method with multi-class tabu lists stores divergent Pareto optima in the long-term tabu lists. Using the intrinsic property of divergent search ability of the method, it also searches for the better Pareto solutions.

Figure 9 shows the results of the proposed method and the conventional method with population ranking. The figure suggests that Tabu-GA generates the better solutions. The upper graph of Figure 9 shows the case with the length 5 long-term tabu lists and the lower graph of the figure shows the case where the length of the long term tabu lists is 20. Each iteration is 300. The figure suggests that we can control the frontier line of Pareto optima by changing the length of the tabu lists.

4.7 RESULTS OF EXPERIMENTS

Table 3 shows the results of some multimodal experiments, where SGA is Simple-GA, three values are Best/Ave/Success, Best is the best iteration in ten executions, Ave is the average iteration of successful executions in them, and Success is the number of successful executions in them. Although the simple GA with elite seems better than Tabu-GA to solve FMS parameters function, it fails at eight times in ten executions to get the optimum solution, while Tabu-GA succeeds at all times to do it.

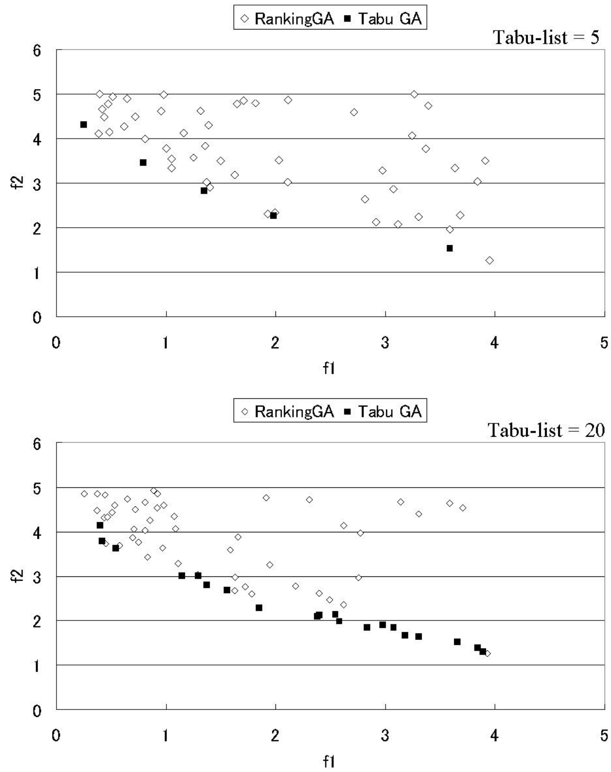

Figure:9 Multiobjective optimization

Table 3: Results of Multimodal Experiments.

Function	SGA	SGA with elite	Tabu-GA
Rastrigin(n=2)	829/829/1	141/497/5	73/264/10
Rastrigin(n=5)	-/-/0	-/-/0	2380/3253/7
Rastrigin(n=10)	-/-/0	-/-/0	3621/4854/5
FMS-parameters	-/-/0	126/282/2	143/608/10

Table 4 shows the results of some multiobjective experiments, where Pareto is the number of Pareto optima and (n) is the tabu list size. While Ranking Selection GA(Fonseca.1993) gets six Pareto solutions on the frontier line of Pareto optima, Tabu-GA can get flexible and diverse Pareto solutions depending on a tabu list size.

Table 4: Results of Multiobjective Experiments.

Method	Pareto
Ranking Selection	6
Tabu-GA (5)	5
Tabu-GA (10)	9
Tabu-GA (20)	16

5 RELATED WORK

The hybridization of Genetic Algorithms with simulated annealing, tabu search, artificial neural networks, and expert system aims at improving the performance of the searching capabilities to difficult problems (Costa, 1995; Glover, 1994; Glover, Kelly, Laguna, 1995; Kitano, 1990; Malek, Guruswamy, 1989; Mantawy, Abdel-Magid, Selim, 1999; Muhlenbein, Gorges-scheleuter, Kramer, 1988; Muhlenbein, 1992; Powel, Tong, Skolnick, 1989; Ulder, Aarts, H.Bandelt, Laarhoven, Pasch, 1991). However, most of the studies in the literature have focused on the global search via GAs and the local search via the other methods. Fine tuning of the local search has been the main issue. On the other hand, we will focus on the selection process of GA iteration via multiple tabu lists.

We believe that GAs provide unique general ways to multimodal function Optimization. There also have been various studies (Echelman, Shaffer, 1991; DeJong, Spears, 1989; Goldberg, 1989; Goldberg, 1990; Tsutsui, Fujimoto, 1993; Tsutsui, Fujimoto, 1994). The proposed methods to keep the diversity of the population include (1) sharing methods, which utilize some sharing functions to avoid the convergence of similar individuals, (2) crowding methods, which constrain the replacements of new individuals, and (3) restrictions of crossover operations. However, these methods are difficult to apply to practical problems. (1) Unsuitable sharing functions often prevent individuals exploiting near optimal regions, (2) the crowding often failed to avoid the convergence of the earlier stage of iterations, (3) the method is too artificial for our problems.

About multiobjective function optimization problems, or problems to find Pareto optimal solutions, there have been various work to extend GAs (Cantu-Paz, 1999; Coello, 1999; Fonseca, Fleming, 1993; Hiroyasu, Miki, Watanabe, 1999; Horn, Nafpliotis,1993; Schaffer, 1985; Srinivas, Deb, 1993; Tamaki, Kita, Kobayashi, 1996). These studies include (1) methods to divide individuals into subgroups, each of which corresponds to each objective function, (2) methods to rank Pareto optimal individuals not to be covered by the other individuals, (3) combination of tournament and sharing methods, and (4) methods to divide Pareto solutions to some ranges. The proposed method is a natural and general extension of the conventional methods.

These studies in the literature have common characteristics to improve GAs by adding capabilities of the diversity of populations, local and/or distributed search. In this paper, we will propose a novel uniform method with multiple tabu lists.

6 CONCLUDING REMARKS

This paper has described a novel method to directly store individuals into multiple tabu lists during GA-iterations. Although the basic idea and the algorithm are very simple, the experimental results have suggested that the method is powerful and robust against very wide class of problems. Because the proposed method only focuses on the population diversity and the selection process, and it does not depend on the variety of genetic operations, we will be able to improve the performance of the proposed methods by employing real-coded genetic operations. (Ono, Kita, Kobayashi, 1999) Practically, we have applied the method to very difficult simulation problems: TRURL(Terano, 1998; Kurahashi, 1999), artificial society model for analyzing complex social interaction problems. Our future work include i) to investigate the sensitivity of the parameters of the Tabu-GA, ii) to improve the proposed method to GA hard domains, and iii) to apply it to large scale problems.

References

E. Cantu-Paz (1999). Topologies, Migration Rates, and Multi-Population Parallel Genetic Algorithms, *Proc. of the Genetic and Evolutionary Computation Conference(GECCO'99),* **1**:91-98.

A. Coello (1999). An Updated Survey of Evolutionary Multiobjective Optimization Techniques: State of the Art and Future Trends, *Proc. of Congress on Evolutionary Computaion*:1-11.

D. Costa (1995). An Evolutionary Tabu Search Algorithm and The NHL Scheduling Problem. *INFOR:Information Systems and Operational Research,* **33** (3):161-178.

K. DeJong and W. Spears (1989). Using Genetic Algorithms to Solve NP-complete Problems, *Proc. 3rd ICGA.*

L. Eshelman and J. Shaffer (1991). Preventing Premature Convergence in Genetic Algorithm by Preventing Incest, *Proc. of the Fourth ICGA*:115-122.

C. Fonseca and P. Fleming (1993). Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization, *Proc. of the Fifth International Conference on Genetic Algorithms*:416-423.

F. Glover (1994). Tabu search for nonlinear and parametric optimization(with links to genetic algorithms). *Discrete Applied Mathematics*,**49**:231-255.

F. Glover, J. Kelly and M. Laguna (1995). Genetic Algorithms and Tabu Search:Hybrid for Optimization. *Computers and Operations Research,* **22** (1):111-134.

D. E. Goldberg (1989). Genetic Algorithms in Search. *Optimization and Machine Learning,* Mass:Addison-Wesley.

D. E. Goldberg (1990). A Note on Boltzman Tournament Selection for Genetic Algorothms and Populationoriented Simulated Annealing, *Complex Systems,* **4**:445-460.

T. Hiroyasu, M. Miki and S. Watanabe (1999). Divided Range Genetic Algorithms in Multiobjective Optimization Problems, *Proc of International Workshop on Emergent Synthesis IWES'99*:57-66.

J. Horn and N. Nafpliotis (1993). Multiobjective optimization using the Niched Pareto Genetic Algorithm, *Technical Report IlliGAl Report 93005,* University of Illinois at Urabama-Champaign.

H. Kitano (1990). Empirical Studies on the Speed of Convergence of Neural Network Training using Genetic Algorithms, *Proc. of the Eighth National Conference on Artificial Intelligence* (AAAI-90) **7**.

S. Kurahashi, U. Minami and T. Terano (1999). Why not Multiple Solutions: Agent-Based Social Interaction Analysis via Inverse Simulation, *Proc. of IEEE International Conference on Systems, Man, and Cybernetics* (SMC'99):smc_2048.

M. Malek, M. Guruswamy, et al. (1989). Serial and parallel simulated annealing and tabu search algorithms for the travelling salesman problem, *Annals of Operations Research*, **21**:59-84.

A. Mantawy, Y. Abdel-Magid and S. Selim(1999).A New Genetic-based Tabu Search Algorithm for Unit Commitment Problem, *Electric Power Systems Research,* **49**:71-78.

H. Muhlenbein, M. Gorges-scheleuter and O. Kramer (1988). Evolution algorithms in combinatorial optimization, *Parallel Computing*, **7**:65-85.

H. Muhlenbein (1992). Parallel Genetic Algorithm in Combinatorial Optimization, *Computer Science and Operations Research*, Pergamon Press.:441-456 .

I. Ono, H. Kita and S. Kobayashi (1999). A Robust Real-Coded Genetic Algorithm using Unimodal Normal Distribution Crossover Augmented by Uniform Crossover : Effets of Self-Adaptaion of Crossover Probabilies, *Proc. of the Genetic and Evolutionary Computation Conference(GECCO'99),* **1**:496-503.

D. Powell, S. Tong and M. Skolnick (1989). EnGENEous: Domain Independent Machine Learning for Design Optimization, *Proc. of the Third ICGA*:151-159.

D. Schaffer (1985). Multiple objective optimization with vector evaluated genetic algorithms, *Proc. of the First International Conference on Genetic Algorithms*:93-100.

N. Srinivas and K. Deb (1993). Multiobjective optimization using nondominated sorting in genetic algorithms, *Technical report, Department of Mechanical Engineering*, Indian Institute of Technology, Kanput, India.

H. Tamaki, H. Kita and S. Kobayashi (1996). Multi-Objective Optimization by Genetic Algorithms. *Proc. of the 1996 International Conference on Evolutionary Computation(ICEC'96)*:517-522.

T. Terano, S. Kurahashi, U. Minami (1998). TRURL: Artificial World for Social Interaction Studies. *Artificial Life VI*:326-335.

S. Tsutsui and Y. Fujimoto (1993). Forking Genetic Algorithm with Blocking and Shrinking Modes. *Proc. of the Fifth ICGA*:206-213.

S. Tsutsui and Y. Fujimoto (1994). Extended Forking Genetic Algorithm for Order Representation. *Proc. of the First IEEE Conf. on Evolutionary Computation*:170-175.

N.Ulder, E.Aarts, H.Bandelt, P. Laarhoven and E. Pasch (1991). Genetic Local Search Algorithms for the Traveling Salesman Problem. *Parallel Problem-solving from Nature* Lecture Notes in Computer Science 496:109-116.