Schema Theorems without Expectations

Riccardo Poli
School of Computer Science
The University of Birmingham
Birmingham, B15 2TT, UK
R.Poli@cs.bham.ac.uk

Abstract

This paper presents two new schema theorems in which expectations are absent. The first theorem provides confidence intervals for the number of instances of a schema at the next generation. The second gives a lower bound for the same quantity.

1 THEOREMS

In (Poli *et al.* 1998) we noted that the process of propagation of a schema H from generation t to generation $t+1$ can be seen as a Bernoulli trial with success probability $\alpha(H,t)$, where $\alpha(H,t)$ is the probability that H either survives or is created after selection, crossover and mutation. Thus, the number of instances of H at generation $t+1$, $m(H,t+1)$, is binomially distributed. So, given $\alpha(H,t)$ we can calculate exactly the probability, $\Pr\{m(H,t+1) \geq x\}$, that the schema H will have at least x instances at generation $t+1$, for any given x. Unfortunately, the result of this calculation is difficult to use (Poli 1999).

One way to remove this problem is to *not* fully exploit our knowledge about the probability distribution of $m(H,t+1)$ when computing $\Pr\{m(H,t+1) \geq x\}$. Instead we could use Chebyshev's inequality: $\Pr\{|X-\mu| < k\sigma\} \geq 1 - \frac{1}{k^2}$ where X is a stochastic variable (with *any* probability distribution), μ is the mean of X, σ is its standard deviation and k is an arbitrary positive number (Spiegel 1975).

Since $m(H,t+1)$ is binomially distributed, $\mu = M\alpha$ and $\sigma = \sqrt{M\alpha(1-\alpha)}$, where α is a shorthand notation for $\alpha(H,t)$ and M is the population size. By substituting these equations into Chebyshev's inequality we obtain:

Theorem 1 (Two-sided Probabilistic Schema Theorem). *For any given constant* $k > 0$

$$\Pr\{|m(H,t+1) - M\alpha| \leq k\sqrt{M\alpha(1-\alpha)}\} \geq 1 - \frac{1}{k^2}.$$

Also, since $\Pr\{m(H,t+1) > M\alpha - k\sqrt{M\alpha(1-\alpha)}\} \geq \Pr\{|m(H,t+1) - M\alpha| \leq k\sqrt{M\alpha(1-\alpha)}\}$, we obtain

Theorem 2 (Probabilistic Schema Theorem). *For any given constant* $k > 0$

$$\Pr\{m(H,t+1) > M\alpha - k\sqrt{M\alpha(1-\alpha)}\} \geq 1 - \frac{1}{k^2}.$$

As discussed in (Poli 1999), this schema theorem can be used to predict the past from the future. This allows one to find the conditions (at the previous generation) under which solutions will be found at a given generation. By using the idea of discovering one new bit of the solution per generation and by recursively applying the schema theorem to such conditions one can find under which conditions on the initial generation the GA will converge in a constant time.

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